

Homework #3, PHY 674, 8 September 1995

- (X12). Let $G = \text{SO}(n)$ be the Lie group of all orthogonal $n \times n$ matrices with real coefficients and determinant 1 (one). (A matrix with real coefficients is called **orthogonal** if its inverse is equal to its transpose. It is called **special**, if its determinant is one.) Why do these matrices form a group under matrix multiplication? I want a few arguments, not pages of proof. (2 points)
- (X13). The Lie algebra $LG = \mathfrak{so}(n)$ of the group $\text{SO}(n)$ is the tangent space of the group G at the neutral element (matrix with all ones in the diagonal). The exponential map

$$\exp : LG \rightarrow G \quad (1)$$

$$A \mapsto \exp(iA) = \sum_{\nu=0}^{\infty} \frac{1}{\nu!} (iA)^{\nu} \quad (2)$$

is the local chart which makes the group G (near the neutral element) and the Lie Algebra LG (near the zero vector) look alike. Using this local chart, find out as much as you can about the Lie algebra LG : What kind of matrices are in LG ? What is the dimension of LG as a real vector space? (4 points).

- (X14). Using the same exponential map, calculate the Lie algebra $\mathfrak{su}(n)$ for the Lie group $G = \text{SU}(n)$ of $n \times n$ special unitary matrixes with complex coefficients. A complex matrix A is called **unitary**, if the transpose conjugate of A is equal to the inverse of A . It is called **special**, if the value of the determinant is 1. Calculate the dimension of $\mathfrak{su}(n)$ as a real vector space. For which n do $\mathfrak{su}(n)$ and $\mathfrak{so}(n)$ have the same dimension? (4 points)

- (X15). For every matrix $A \in LG$, we can define a curve α^A in G :

$$\alpha^A : \mathbb{R} \rightarrow G \quad (3)$$

$$t \mapsto \alpha^A(t) = \exp(itA) = \sum_{\nu=0}^{\infty} \frac{1}{\nu!} (itA)^{\nu} \quad (4)$$

This curve is called a one-parameter group. Show that this map is a homomorphism of groups. (4 points) Hint: You probably did not work out this nasty product of exponentials in the last problem sets, so why do it now?

Due Date:
Monday, 18 September 1995, 2:10 pm